Study Guide for Complex Analysis Exam

I. Calculus and Undergraduate Analysis

Continuity and differentiation in one and several real variables
Inverse and implicit function theorems
Compactness and connectedness in analysis
Uniform convergence and uniform continuity
Riemann integrals
Contour integrals and Green's theorem
Reference: [3].

II. Preliminary Topics in Complex Analysis

Complex arithmetic
Analyticity, harmonic functions, and the Cauchy-Riemann equations
Contour Integration in **C**
References: [1] Chapters 1, 2; [2] Chapters 1, 2, 4; [4] Chapter 1.

III. Cauchy's Theorem and its consequences

Cauchy's theorem and integral formula, Morera's theorem, Schwarz reflection
Uniform convergence of analytic functions
Taylor and Laurent expansions
Maximum modulus principle and Schwarz's lemma
Liouville's theorem and the Fundamental theorem of algebra
Residue theorem and applications
Singularities and meromorphic functions, including the Casorati-Weierstrass theorem
Rouche's theorem, the argument principle, and the open mapping theorem
Estimates using Cauchy Integral Formula: Cauchy inequalities and, more generally, bounds on holomorphic functions and their derivatives on compact sets
References: [1] Chapters 4, 5, 6; [2] Chapters 5, 7, 8, 9; [4] Chapters 2, 3, 5, 8 (§2,3).

IV. Conformal Mapping

General properties of conformal mappings
Analytic and mapping properties of linear fractional transformations
Automorphisms of the disk, plane, and Riemann sphere
References: [1] Chapters 3, 8; [2] Chapters 3, 4; [4] Chapter 8 (§1,2).

References

[1] L. Ahlfors, *Complex Analysis*, Third Edition, McGraw-Hill.
[2] E. Hille, *Analytic Function Theory*, Vol. 1, Ginn and Company.
[3] W. Rudin, *Principles of Mathematical Analysis*, Third Edition, McGraw-Hill.
[4] E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press.

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