# CHAPTER 2 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Consider the sequence $(x_n)$ whose terms are given by the formula

$$x_n = \frac{(\cos n\pi)(\sin^2 n)}{\sqrt[e]{n}}$$

for each integer $n \geq 1$. Given that this sequence converges, what is its limit?

(A) 0 (B) 1 (C) log 2 (D) $\sqrt[e]{2}$ (E) $\sqrt[e]{e}$

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2. Let $(x_n)$ be the sequence with $x_1 = 2$ and $x_n = \sqrt{5x_{n-1} + 6}$ for every integer $n \geq 2$. Given that this sequence converges, what is its limit?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 16

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3. Let $[x]$ denote the greatest integer $\leq x$. If $n$ is a positive integer, then

$$\lim_{x \to -n-} \left(|x| - [x]\right) - \lim_{x \to n-} \left(|x| - [x]\right) = ?$$

(A) –2 (B) 0 (C) 2 (D) $2n - 1$ (E) $2n$

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4. Evaluate the following limit:

$$\lim_{x \to 0} \frac{\arcsin x - x}{x^3}$$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 1

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5. The curve whose equation is

$$2x^2 + 3x - 2xy - y = 6$$

has two asymptotes. Identify these lines.

(A) $x = -1$ and $y = -2$ (B) $x = -2$ and $y = 1$ (C) $x = -\frac{1}{2}$ and $y = x$
(D) $x = -\frac{1}{2}$ and $y = x + 1$ (E) $x = \frac{1}{2}$ and $y = 1 - x$

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6. If the function

$$f(x)=\begin{cases}\dfrac{x^2-6x+8}{x^3-2x^2+2x-4} & \text{if } x\neq 2\\ k & \text{if } x=2\end{cases}$$

is continuous everywhere, what is the value of $k$?

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{3}$ (E) –1

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7. Evaluate the following limit:

$$\lim_{x\to 0}\left[\frac{1}{x^2}\int_0^x \frac{t+t^2}{1+\sin t}\,dt\right]$$

(A) $\frac{1}{2\pi}$ (B) $\frac{1}{\pi}$ (C) $\frac{1}{2}$ (D) 1 (E) $\frac{\pi}{2}$

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8. Determine the domain of the following function:

$$f(x)=\arcsin(\log\sqrt{x})$$

(A) $[0, \frac{1}{e^2}]$ (B) $[\frac{1}{e^2}, 1]$ (C) $[e, e^2]$ (D) $[\frac{1}{e^2}, e^2]$ (E) $[1, e^2]$

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9. Evaluate the derivative of the following function at $x = e$:

$$f(x)=\arcsin(\log\sqrt{x})$$

(A) $\frac{1}{e\sqrt{3}}$ (B) $\frac{e}{\sqrt{2}}$ (C) $\frac{\pi e}{2}$ (D) $\sqrt{2e}$ (E) $\frac{3e}{\sqrt{2}}$

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10. For what values of $m$ and $b$ will the following function have a derivative for every $x$?

$$f(x)=\begin{cases}x^2+x-3 & \text{if } x\leq 1\\ mx+b & \text{if } x>1\end{cases}$$

(A) $m = 3, b = -2$ (B) $m = -2, b = -3$ (C) $m = 1, b = -4$
(D) $m = -2, b = 1$ (E) $m = 3, b = -4$

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11. If $f(x)$ is a function that's differentiable everywhere, what is the value of this limit?

$$\lim_{h\to 0}\frac{f(x+3h^2)-f(x-h^2)}{2h^2}$$

(A) $4f'(x)$ (B) $2f'(x)$ (C) $f'(x)$ (D) $\frac{1}{2}f'(x)$

(E) The limit does not exist.

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12. What is the equation of the tangent line to the curve $y = x^3 - 3x^2 + 4x$ at the curve's inflection point?

(A) $y = 2x - 3$ (B) $y = x - 1$ (C) $y = x + 1$ (D) $y = 3x - 2$ (E) $x + y = 1$

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13. What is the slope of the tangent line to the curve $xy(x + y) = x + y^4$ at the point (1, 1)?

(A) 2 (B) 1 (C) 0 (D) –1 (E) –2

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14. If $f(x)=2|x-1|+(x-1)^2$, what is the value of $f'(0)$?

(A) 4 (B) 2 (C) 0 (D) –2 (E) –4

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15. If

$$f(x)=\frac{e^x \arccos x}{\cos x}$$

then the slope of the line tangent to the graph of $f$ at its $y$-intercept is

(A) $-\frac{\pi}{2}$ (B) –1 (C) $\frac{\pi}{2}-1$ (D) 1 (E) $\frac{\pi}{2}+1$

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16. Let $y=\frac{1}{\sqrt{x^3+1}}$. If $x$ increases from 2 to 2.09, which of the following most closely approximates the change in $y$?

(A) 0.08 (B) 0.04 (C) –0.02 (D) –0.06 (E) –0.09

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17. If $f(1) = 1$ and $f'(1) = -1$, then the value of $\frac{d}{dx}\left[\frac{f(x^3)}{xf(x^2)}\right]$ at $x = 1$ is equal to

(A) 1 (B) 0 (C) –1 (D) –2 (E) –3

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18. If $n$ is a positive integer, what is the value of the $n^{\text{th}}$ derivative of $f(x) = \frac{1}{1-2x}$ at $x = -\frac{1}{2}$?

(A) $\frac{1}{2}(n^n)$ (B) $\frac{1}{2}(n!)$ (C) $\frac{1}{2}n$ (D) $n$ (E) $\frac{n^n}{n!}$

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19. Let $f(x)$ be continuous on a bounded interval, $[a, b]$, where $a \neq b$, such that $f(a) = 1$ and $f(b) = 3$, and $f'(x)$ exists for every $x$ in $(a, b)$. What does the Mean-Value theorem say about $f$?

(A) There exists a number $c$ in the interval $(a, b)$ such that $f'(c) = 0$.
(B) There exists a number $c$ in the interval $(a, b)$ such that $f(c) = 0$.
(C) There exists a number $c$ in the interval $(a, b)$ such that $f'(c) = 2$.
(D) There exists a number $c$ in the interval $(a, b)$ such that $f'(c) = 2(b-a)$.
(E) There exists a number $c$ in the interval $(a, b)$ such that $(b-a)f'(c) = 2$.

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20. What is the maximum area of a rectangle inscribed in a semicircle of radius $a$?

(A) $\frac{\sqrt{2}}{2}a^2$ (B) $\frac{\sqrt{3}}{2}a^2$ (C) $a^2$ (D) $\frac{\pi}{2\sqrt{2}}a^2$ (E) $a^2\sqrt{2}$

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21. The following function is defined for all positive $x$:

$$f(x) = \int_x^{2x} \frac{\sin t}{t}\,dt$$

At what value of $x$ on the interval $(0, \frac{3\pi}{2})$ does this function attain a local maximum?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\pi$ (E) $\frac{2\pi}{3}$

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22. Let $f(x) = x^k e^{-x}$, where $k$ is a positive constant. For $x > 0$, what is the maximum value attained by $f$?

(A) $\left(\frac{e}{k}\right)^k$ (B) $\sqrt[k]{\frac{e}{k^k}}$ (C) $\frac{(\log k)^k}{k}$ (D) $\left(\frac{e}{\log k}\right)^k$ (E) $\left(\frac{k}{e}\right)^k$

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23. The radius of a circle is decreasing at a rate of 0.5 cm per second. At what rate, in cm$^2$/sec, is the circle's area decreasing when the radius is 4 cm?

(A) $4\pi$ (B) $2\pi$ (C) $\pi$ (D) $\frac{1}{2}\pi$ (E) $\frac{1}{4}\pi$

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24. The function $f(x) = \int_{e^x}^{e^{2x}} t \log t\,dt$ has an absolute minimum at $x = 0$, and a local maximum at $x =$

(A) $-\log 4$ (B) $-\log 2$ (C) $\log 2$ (D) 1 (E) $\log 4$

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**25.** Evaluate the following integral:

$$\int_{-1}^{0} x^2(x+1)^3\,dx$$

(A) $-\frac{7}{20}$ (B) $-\frac{1}{60}$ (C) $\frac{2}{15}$ (D) $\frac{1}{60}$ (E) $\frac{7}{20}$

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**26.** If $[x]$ denotes the greatest integer $\leq x$, then $\int_0^{\frac{7}{2}} [x]dx =$

(A) $\frac{5}{2}$ (B) $\frac{7}{2}$ (C) $\frac{9}{2}$ (D) $\frac{17}{2}$ (E) $\frac{37}{2}$

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**27.** If

$$f(x)=\begin{cases}-2(x+1) & \text{if } x\leq 0\\ k(1-x^2) & \text{if } x>0\end{cases}$$

then the value of $k$ for which $\int_{-1}^{1} f(x)dx = 1$ is

(A) –1 (B) 0 (C) 1 (D) 2 (E) 3

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**28.** Integrate $\int \frac{x^2\,dx}{\sqrt{1-x^2}}$.

(A) $\frac{1}{2}\left(\arcsin x-\sqrt{1-x^2}\right)+c$ (B) $\frac{1}{2}\left(\arcsin x+x\sqrt{1-x^2}\right)+c$ (C) $\frac{1}{2}\left(x\arcsin x-\sqrt{1-x^2}\right)+c$

(D) $\frac{1}{2}\left(\arcsin x-x\sqrt{1-x^2}\right)+c$ (E) $\frac{1}{2}\left(x\arcsin x+\sqrt{1-x^2}\right)+c$

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**29.** What is the area of the region in the first quadrant bounded by the curve $y = x \arctan x$ and the line $x = 1$?

(A) $\frac{\pi-4}{4}$ (B) $\frac{\pi-2}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi+2}{4}$ (E) $\frac{\pi+4}{4}$

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**30.** Simplify the following:

$$\exp\int_3^5 \frac{dx}{x^2-3x+2}$$

[Note: Recall that exp $x$ is a standard, alternate notation for $e^x$.]

(A) $\frac{3}{8}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) $\frac{5}{3}$

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31. Calculate the area of the region in the first quadrant bounded by the graphs of $y = 8x$, $y = x^3$, and $y = 8$.

(A) 12 (B) 8 (C) 6 (D) $\frac{16}{3}$ (E) 4

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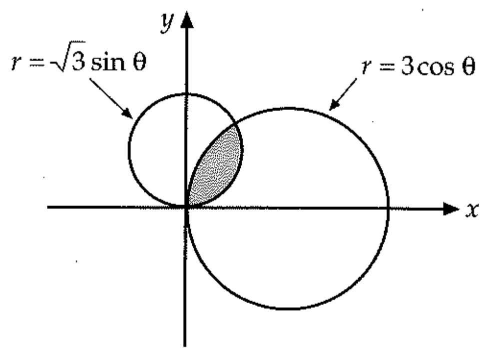
32. Which of the following expressions gives the area of the region bounded by the two circles pictured below?




(A) $\int_0^{\frac{\pi}{2}} \frac{1}{2}\left[(\sqrt{3}\sin\theta)^2 - (3\cos\theta)^2\right] d\theta$

(B) $\int_0^{\frac{\pi}{6}} \frac{1}{2}(3\cos\theta)^2\, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(\sqrt{3}\sin\theta)^2\, d\theta$

(C) $\int_0^{\frac{\pi}{6}} \frac{1}{2}(\sqrt{3}\sin\theta)^2\, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(3\cos\theta)^2\, d\theta$

(D) $\int_0^{\frac{\pi}{3}} \frac{1}{2}(3\cos\theta)^2\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(\sqrt{3}\sin\theta)^2\, d\theta$

(E) $\int_0^{\frac{\pi}{3}} \frac{1}{2}(\sqrt{3}\sin\theta)^2\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(3\cos\theta)^2\, d\theta$

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33. Let $a$ and $b$ be positive numbers. The region in the second quadrant bounded by the graphs of $y = ax^2$ and $y = -bx$ is revolved around the $x$-axis. Which of the following relationships between $a$ and $b$ would imply that the volume of this solid of revolution is a constant, independent of $a$ and $b$?

(A) $b^4 = 2a^5$ (B) $b^3 = 2a^5$ (C) $b^5 = 2a^3$ (D) $b^4 = 2a^2$ (E) $b^2 = 2a^3$

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34. The region bounded by the graphs of $y = x^2$ and $y = 6 - |x|$ is revolved around the $y$-axis. What is the volume of the generated solid?

(A) $\frac{32}{3}\pi$ (B) $9\pi$ (C) $8\pi$ (D) $\frac{20}{3}\pi$ (E) $\frac{16}{3}\pi$

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35. Calculate the length of the portion of the hypocycloid $x^{2/3} + y^{2/3} = 1$ in the first quadrant from the point $\left(\frac{1}{8}, \frac{3\sqrt{3}}{8}\right)$, to the point $(1, 0)$.

(A) $\frac{9}{8}$ (B) $\frac{3\sqrt{2}}{4}$ (C) 1 (D) $\frac{5\sqrt{2}}{8}$ (E) $\frac{\sqrt{3}}{2}$

36. What positive value of $a$ satisfies the following equation?

$$\int_e^{a^e} \frac{dx}{x\int_a^{ax} \frac{dy}{y}} = 1$$

(A) $\frac{1}{e}$ (B) $\sqrt[e]{e}$ (C) $\sqrt{e}$ (D) $e$ (E) $e^2$

37. Evaluate the following limit:

$$\lim_{x \to 0} (\cos x)^{\cot^2 x}$$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{e}}$ (C) $\frac{\sqrt{e}}{2}$ (D) 1 (E) $\sqrt{e}$

38. Let $n$ be a number for which the improper integral

$$\int_e^{\infty} \frac{dx}{x(\log x)^n}$$

converges. Determine the value of the integral.

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n}$ (C) $\frac{1}{n-1}$ (D) $\frac{\log n}{n+1}$ (E) $\frac{\log n}{n-1}$

39. Find the positive value of $a$ that satisfies the equation:

$$\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \int_0^a \frac{x\,dx}{\sqrt{a^2 - x^2}}$$

(A) $\frac{2\sqrt{2}}{\pi}$ (B) 1 (C) $\frac{\pi}{2\sqrt{2}}$ (D) $\sqrt{2}$ (E) $\frac{\pi}{2}$

**40.** Which of the following improper integrals converge?

I. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$

II. $\displaystyle\int_1^{\infty} xe^{-x}\,dx$

III. $\displaystyle\int_0^{2} \frac{dx}{(2-x)^2}$

(A) I only (B) I and II only (C) II only
(D) I and III only (E) II and III only

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**41.** Which of the following infinite series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^4(\arctan n)}{n\sqrt[4]{n}}$

II. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\log n}$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)^3}{5(n+2)(n+3)(n+4)}$

(A) I only (B) I and II only (C) II only
(D) I and III only (E) II and III only

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**42.** Find the smallest value of $b$ that makes the following statement true:

If $0 \le a < b$, then the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 a^n}{(2n)!}$ converges.

(A) 1 (B) 2 log 2 (C) 2 (D) $\sqrt{2}$ (E) 4

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**43.** Evaluate the following limit:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{k}{n^2} - \frac{k^2}{n^3}\right]$$

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ (E) $\frac{1}{12}$

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**44.** Which of the following statements are true?

I. If $a_n \geq 0$ for every $n$, then: $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow$ $\sum_{n=1}^{\infty} \sqrt{a_n}$ converges.

II. If $a_n \geq 0$ for every $n$, then: $\sum_{n=1}^{\infty} na_n$ converges $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ converges.

III. If $a_n \geq 0$ and $a_{n+1} \leq a_n$ for every $n$, then: $\sum_{n=1}^{\infty} a_n^2$ converges $\Rightarrow$ $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(A) I and II only (B) I and III only (C) II only
(D) II and III only (E) III only

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**45.** If $-1 < x < 1$, then $\sum_{n=1}^{\infty} nx^{2n} =$

(A) $\frac{x^3}{(1-x)^2}$ (B) $\frac{x^2}{(1-x^2)^2}$ (C) $\frac{x}{(1+x^2)^2}$
(D) $\frac{x^3}{(1+x)^2}$ (E) $\frac{x^2}{(1+x^2)^2}$

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**46.** The smallest positive integer $x$ for which the power series $\sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!} x^n$ does *not* converge is

(A) 4 (B) 6 (C) 7 (D) 8 (E) 9

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**47.** In the Taylor series expansion (in powers of $x$) of the function $f(x) = e^{x^2-x}$, what is the coefficient of $x^3$?

(A) $-7$ (B) $-\frac{3}{2}$ (C) $-\frac{7}{6}$ (D) $\frac{7}{6}$ (E) $\frac{3}{2}$

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**48.** If $k_i$ $(i = 0, 1, 2, 3, 4)$ are constants such that $x^4 = k_0 + k_1(x+1) + k_2(x+1)^2 + k_3(x+1)^3 + k_4(x+1)^4$ is an identity in $x$, what is the value of $k_3$?

(A) $-4$ (B) $-3$ (C) $-2$ (D) 3 (E) 4

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