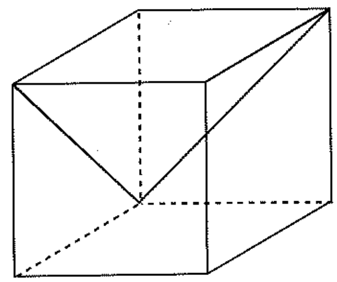
CHAPTER 3 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Find the angle between the diagonals of the back and left faces of the cube shown below:



(A) $60°$ (B) $\cos^{-1}\frac{1}{\sqrt{6}}$ (C) $\cos^{-1}\frac{1}{3\sqrt{2}}$ (D) $90°$ (E) $120°$

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2. If $\hat{\mathbf{i}} = (1, 0, 0)$, $\hat{\mathbf{j}} = (0, 1, 0)$, and $\hat{\mathbf{k}} = (0, 0, 1)$, which one of the following vectors is *not* orthogonal to $\mathbf{v} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$?

(A) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (B) $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (C) $3\hat{\mathbf{i}} - 2\hat{\mathbf{k}}$ (D) $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (E) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

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3. What's the area of the triangle whose vertices are (0, 0, 1), (0, 2, 0), and (3, 0, 0)?

(A) $\frac{8}{3}$ (B) $\frac{7}{2}$ (C) 3 (D) 6 (E) 7

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4. Given the vector identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$, which of the following $\mathbf{V}$ satisfies the equation $\mathbf{A} \times \mathbf{V} = \mathbf{B}$, where $\mathbf{A}$ is a unit vector and $\mathbf{B}$ is a vector orthogonal to $\mathbf{A}$?

(A) $\mathbf{B} + (\mathbf{A} \times \mathbf{B})$ (B) $\mathbf{B} - (\mathbf{A} \times \mathbf{B})$ (C) $\mathbf{A} \times \mathbf{B}$
(D) $\mathbf{A} + (\mathbf{A} \times \mathbf{B})$ (E) $\mathbf{A} - (\mathbf{A} \times \mathbf{B})$

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5. Let L be the line in space that passes through the points $P = (-1, -2, 4)$ and $Q = (4, 2, 1)$. At what point does L intersect the plane $x + y + 2z = 11$?

(A) (5, –4, 1) (B) (1, 2, 4) (C) (8, 9, –3) (D) (6, 7, –1) (E) (9, 6, –2)

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6. Let $P = (1, -1, 1)$ and $Q = (-3, 3, 3)$. Find the point R on the line containing P and Q whose distance from P is 3 times its distance from Q, given that R is *not* between P and Q.

(A) $(-4, 4, \frac{7}{2})$ (B) $(-5, 5, 4)$ (C) $(-\frac{13}{3}, \frac{13}{3}, \frac{11}{3})$
(D) $(-\frac{15}{2}, \frac{15}{2}, \frac{11}{2})$ (E) $(-9, 9, 6)$

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7. If L is the line through the point $A = (3, 2, 1)$ and parallel to the vector $\mathbf{v} = (-2, 1, 3)$, what's the equation of the plane that contains L and the point $B = (-2, 3, 1)$?

(A) $-x + y + z = 6$
(B) $3x - 2y - z = 4$
(C) $x + 6y - 11z = 5$
(D) $x + 5y - z = 12$
(E) $2x + 10y - 19z = 7$

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8. Find the perpendicular distance from the origin to the plane $x + 2y + 2z = 6$.

(A) 1
(B) $\frac{4}{3}$
(C) 2
(D) $\frac{8}{3}$
(E) 3

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9. If the curve $z = f(x)$ in the xz-plane is revolved around the x-axis, which of the following is an equation that describes the resulting surface?

(A) $y^2 + z^2 = |f(x)|$
(B) $z^2 = f(x^2 + y^2)$
(C) $y^2 = f(x^2 + z^2)$
(D) $x^2 + z^2 = [f(x)]^2$
(E) $y^2 + z^2 = [f(x)]^2$

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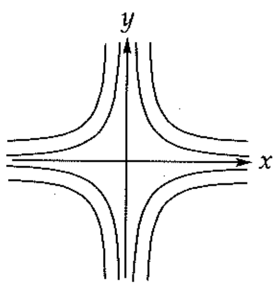
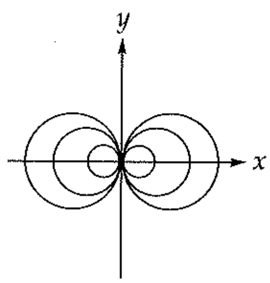
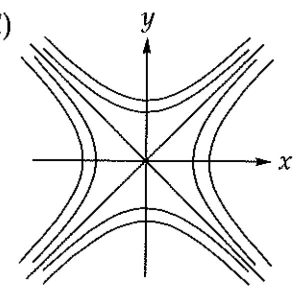
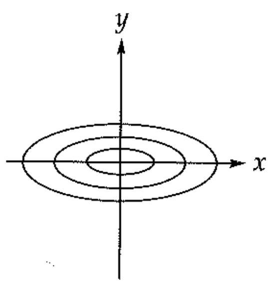
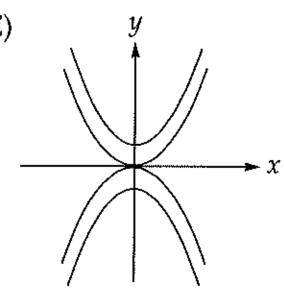
10. Which of the following depicts level curves of the surface whose equation in cylindrical coordinates is $z = r^2 \cos 2\theta$?

(A)


(B)


(C)


(D)


(E)


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11. Consider the following three functions, each of which is defined for all (x, y) in the plane:

$$f_1(x,y)=\begin{cases}\dfrac{x-y}{x+y} & \text{if } x+y\neq 0\\ 1 & \text{if } x+y=0\end{cases}\quad f_2(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}\quad f_3(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

Which of these functions is/are continuous at the origin?

(A) None (B) f_1 only (C) f_2 only (D) f_3 only (E) All three

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12. The plane $y = 1$ slices the surface

$$z=\arctan\frac{x+y}{1-xy}$$

in a curve, C. Find the slope of the tangent line to C at the point where $x = 2$.

(A) –3 (B) –1 (C) $\frac{1}{5}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

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13. If the variables P, V, and T are related by the equation $PV = nRT$, where n and R are constants, simplify the expression

$$\frac{\partial V}{\partial T}\bullet\frac{\partial T}{\partial P}\bullet\frac{\partial P}{\partial V}$$

(A) –1 (B) 1 (C) $-nR$ (D) nR (E) $\frac{1}{nR}$

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14. Let f be the function defined for all (x, y) as follows:

$$f(x,y)=\begin{cases}xy\dfrac{x^2-y^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

What is the value of f_{xy} at the point (0, 0)?

(A) –1 (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) Undefined

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15. A right circular cylinder has base radius $r = 100$ cm and height $h = 100$ cm. Which of the following best describes how the volume of the cylinder will change if r increases to 101 cm and h decreases to 99 cm?

(A) Volume will decrease by approximately $3\pi(100)^2$ cubic cm
(B) Volume will decrease by approximately $\pi(100)^2$ cubic cm
(C) Volume will increase by approximately $\pi(100)^2$ cubic cm
(D) Volume will increase by approximately $2\pi(100)^2$ cubic cm
(E) Volume will increase by approximately $3\pi(100)^2$ cubic cm

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16. Let P be the tangent plane to the surface

$$y^2z - 2xz^2 + 3x^2y = 2$$

at the point $Q = (1, 1, 1)$. Which of the following points also lies in P?

(A) $(4, 5, -3)$ (B) $(6, -4, 3)$ (C) $(3, -1, 5)$ (D) $(5, 3, -2)$ (E) $(-2, 4, 2)$

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17. The equation $x^3z^5 - y^2z^3 - 3xy = 1$ defines an implicit function $z = f(x, y)$. What's the value of $\frac{\partial f}{\partial y}$ at the point $(x, y) = (-1, 1)$?

(A) -8 (B) -1 (C) $-\frac{1}{8}$ (D) $\frac{1}{8}$ (E) 8

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18. Let f, g, and h be functions of two variables that are differentiable everywhere such that $z = f(x, y)$, where $x = g(u, v)$ and $y = h(u, v)$. When $u = 0$ and $v = 1$, the values of x and y are 2 and 1, respectively. Let P_0 denote the point $(u, v) = (0, 1)$, and let Q_0 denote the point $(x, y) = (2, 1)$. Given the following data,

$$\left.\frac{\partial f}{\partial x}\right|_{Q_0} = 11, \quad \left.\frac{\partial f}{\partial y}\right|_{Q_0} = -3, \quad \left.\frac{\partial g}{\partial u}\right|_{P_0} = 1, \quad \left.\frac{\partial h}{\partial u}\right|_{P_0} = -3, \quad \left.\frac{\partial g}{\partial v}\right|_{P_0} = \left.\frac{\partial h}{\partial v}\right|_{P_0} = 2$$

what's the value of $\frac{\partial z}{\partial v}$ at P_0?

(A) -21 (B) 16 (C) 15 (D) 12 (E) -10

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19. The temperature at each point (x, y, z) in a room is given by the equation $T(x, y, z) = 9x^2 - 3y^2 + 6xyz$. A fly is currently hovering at the point $(2, 2, 2)$. In the direction of which of the following vectors should the fly move in order to cool off as rapidly as possible?

(A) $-5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (B) $-4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (C) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ (D) $-2\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ (E) $-6\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

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20. Let $f(x, y)$ be a function that is differentiable everywhere. At a certain point P in the xy-plane, the directional derivative of f in the direction of $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is $\sqrt{2}$ and the directional derivative of f in the direction of $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is $3\sqrt{2}$. What is the maximum directional derivative of f at P?

(A) $3\sqrt{2}$ (B) $2\sqrt{5}$ (C) $43\sqrt{2}$ (D) 6 (E) 8

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21. Which of the following vectors is normal to the surface

$$\log(x + y^2 - z^3) = x - 1$$

at the point where $y = 8$ and $z = 4$?

(A) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (B) $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (C) $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (D) $-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ (E) $\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

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22. The function $f(x, y) = x^3 + y^3 - 3xy$ has a local minimum at exactly one point, P. What's the value of f at P?

(A) –6 (B) –3 (C) –2 (D) –1 (E) 0

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23. Find the minimum distance from the origin to the curve $3x^2 + 4xy + 3y^2 = 20$.

(A) 1 (B) $3\sqrt{2}$ (C) 2 (D) $23\sqrt{2}$ (E) $53\sqrt{2}$

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24. A vertical fence is constructed whose base is the curve $y = x\sqrt{x}$, from (0, 0) to (1, 1), and whose height above each point (x, y) along the curve is $x^3 - y^2 + 27$. Find the area of this fence.

(A) $\frac{1}{9}(5\sqrt{5}-2)$ (B) $5\sqrt{13}-6$ (C) $9\sqrt{3}$ (D) $13\sqrt{13}-8$ (E) 27

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25. If $\mathbf{F} = (3y - 2x)\,\hat{\mathbf{i}} + (x^2 + y)\,\hat{\mathbf{j}}$, find the value of $\int_C \mathbf{F} \bullet d\mathbf{r}$, where C is the portion of the parabola $y = x^2$, directed from (–1, 1) to the origin.

(A) –1 (B) 0 (C) 1 (D) 2 (E) 3

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26. Let C be the portion of the astroid $x^{2/3} + y^{2/3} = 1$ from (1, 0) to (0, 1), which can be parameterized by the equations

$$x = \cos^3 t, \quad y = \sin^3 t$$

as t increases from 0 to $\frac{\pi}{2}$. Evaluate the integral:

$$\int_C (y \cos xy - 1)\,dx + (1 + x \cos xy)\,dy$$

(A) –2 (B) –1 (C) 1 (D) $\frac{1}{2}\pi - 1$ (E) 2

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27. Find the volume of the solid in the first octant of xyz-space, bounded below by the coordinate axes and the unit circle, and bounded above by the surface $z = 8xy$.

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

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28. Set up, but do not evaluate, a double integral that gives the volume of the solid bounded above by the elliptic paraboloid $z = 1 - (x^2 + \frac{1}{9}y^2)$ and bounded below by the elliptic cone $z = \sqrt{x^2 + \frac{1}{9}y^2} - 1$.

(A) $4\int_0^1 \int_0^3 \left(2 - \sqrt{x^2 + \frac{1}{9}y^2}\right) dy\,dx$

(B) $\int_{-1}^1 \int_{-3\sqrt{1-x^2}}^{3\sqrt{1-x^2}} \left(2 - \frac{3}{2}\sqrt{x^2 + \frac{1}{9}y^2}\right) dy\,dx$

(C) $2\int_0^1 \int_{-3}^3 \left(x^2 - \frac{1}{9}y^2 - \sqrt{x^2 + \frac{1}{9}y^2}\right) dy\,dx$

(D) $2\int_0^1 \int_{-3}^3 \left(2 - x^2 - \frac{1}{9}y^2 - \sqrt{x^2 + \frac{1}{9}y^2}\right) dy\,dx$

(E) $\int_{-1}^1 \int_{-3\sqrt{1-x^2}}^{3\sqrt{1-x^2}} \left(2 - x^2 - \frac{1}{9}y^2 - \sqrt{x^2 + \frac{1}{9}y^2}\right) dy\,dx$

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29. If a is a positive number, what's the value of the following double integral?

$$\int_0^{2a} \int_{-\sqrt{2ay-y^2}}^{0} \sqrt{x^2 + y^2}\,dx\,dy$$

(A) $\frac{16}{9}a^3$ (B) $\frac{32}{9}a^3$ (C) $\frac{\pi}{2}a^2$ (D) $\frac{8\pi}{3}a^2$ (E) $2a^4$

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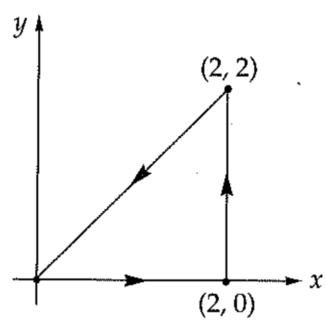
30. Let C be the boundary of the triangular region, as shown below:



Determine the value of the integral:

$$\int_C (e^{2x} - y)\,dx + (2x + y\sqrt{y})\,dy$$

(A) 2 (B) 4 (C) 6 (D) $e^2 + 2\sqrt{2} - 3$ (E) $e^4 + 2(\sqrt{2} - 1)$
