CHAPTER 5 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Two distinct solutions, $\mathbf{x}_1$ and $\mathbf{x}_2$, can be found to the linear system $A\mathbf{x} = \mathbf{b}$. Which of the following is necessarily true?

   (A) $\mathbf{b} = \mathbf{0}$.

   (B) A is invertible.

   (C) A has more columns than rows.

   (D) $\mathbf{x}_1 = -\mathbf{x}_2$.

   (E) There exists a solution $\mathbf{x}$ such that $\mathbf{x} \neq \mathbf{x}_1$ and $\mathbf{x} \neq \mathbf{x}_2$.

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2. The solution of the system

$$\begin{aligned} ax + ay - z &= 1 \\ x - ay - az &= -1 \\ ax - y + az &= 1 \end{aligned}$$

is $(x, y, z) = (a, b, a)$. If a is NOT an integer, what is the numerical value of $a + b$?

(A) $-\frac{3}{2}$ (B) -1 (C) 0 (D) $\frac{1}{2}$ (E) 1

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3. Let A, B, and C be real 2×2 matrices, and let 0 denote the 2×2 zero matrix. Which of the following statements is/are true?

   I. $A^2 = 0 \Rightarrow A = 0$

   II. $AB = AC \Rightarrow B = C$

   III. A is invertible and $A = A^{-1} \Rightarrow A = I$ or $A = -I$

(A) I only (B) I and III only (C) II and III only

(D) III only (E) None of the above

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4. If

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix}$$

then $n =$

(A) -7 (B) -5 (C) 5 (D) 6 (E) 7

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5. If the matrices

$$\begin{pmatrix} 3 & -2 & -2 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix}$$

are inverses of each other, what is the value of c?

(A) –3 (B) –2 (C) 0 (D) 2 (E) 3

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6. The vectors $\mathbf{v}_1 = (-1, 1, 1)$, $\mathbf{v}_2 = (1, 1, 1)$, and $\mathbf{v}_3 = (1, -1, k)$ form a basis for $\mathbf{R}^3$ for all real values of k EXCEPT $k =$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

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7. For the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

let r denote its rank and d denote its determinant. What is the value of $r - d$?

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

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8. If

$$\det \begin{pmatrix} a & b & c \\ k & l & m \\ p & q & r \end{pmatrix} = d$$

then

$$\det \begin{pmatrix} k & 2(a-k) & p+k \\ l & 2(b-l) & q+l \\ m & 2(c-m) & r+m \end{pmatrix} =$$

(A) $-8d$ (B) $-6d$ (C) $-2d$ (D) $2d$ (E) $8d$

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9. For what value of x will the following matrix be noninvertible?

$$\begin{pmatrix} 7 & 6 & 0 & 1 \\ 5 & 4 & x & 0 \\ 8 & 7 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

(A) –1 (B) 0 (C) 1 (D) 3 (E) 9

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10. For what value of d is the vector $\mathbf{b} = (12, 11, d)^T$ in the column space of this matrix?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

(A) –26 (B) 10 (C) 13 (D) –10 (E) 26

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11. What is the dimension of the following subspace of $\mathbf{R}^5$?

$$\text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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12. A square matrix A is said to be **symmetric** if it equals its own transpose: $A = A^T$. What is the dimension of the subspace $S_{n \times n}(\mathbf{R})$ of real symmetric $n \times n$ matrices in the space of all real $n \times n$ matrices, $M_{n \times n}(\mathbf{R})$?

(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n(n-1)$ (C) $\frac{1}{2}n^2$ (D) $\frac{1}{2}n(n+1)$ (E) $\frac{1}{2}n!$

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13. The set of all points (x, y) in $\mathbf{R}^2$ that satisfy the equation

$$\begin{vmatrix} x & y & 1 \\ 0 & y_1 & 1 \\ 1 & y_2 & 1 \end{vmatrix} = 0$$

forms

(A) an ellipse centered at the origin

(B) a line with slope $\frac{y_2}{y_1}$

(C) a circle with center $(0, y_1)$ and radius 1

(D) a line with slope $y_2 - y_1$

(E) a parabola with vertex $(1, y_2)$

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14. The linear transformation T: $\mathbf{R}^2 \to \mathbf{R}^2$ that maps (1, 2) to (–1, 1) and (0, –1) to (2, –1) will map (1, 1) to

(A) (1, 2) (B) (1, 0) (C) (2, –1) (D) (2, 1) (E) (1, 1)

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15. Let T: $\mathbf{R}^5 \to \mathbf{R}^3$ be a linear transformation whose kernel is a three-dimensional subspace of $\mathbf{R}^5$. The set $\{T(\mathbf{x}): \mathbf{x} \in \mathbf{R}^5\}$ is

(A) the trivial subspace

(B) a line through the origin

(C) a plane through the origin

(D) all of $\mathbf{R}^3$

(E) Cannot be determined from the information given

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16. Define linear operators S and T on the xy-plane ($\mathbf{R}^2$) as follows: S rotates each vector 90° counterclockwise, and T reflects each vector through the y-axis. If ST and TS denote the compositions $S \circ T$ and $T \circ S$, respectively, and I is the identity map, which of the following is true?

(A) $ST = I$ (B) $ST = -I$ (C) $TS = I$ (D) $ST = TS$ (E) $ST = -TS$

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17. Choose a nonzero vector $\mathbf{v} = (a, b, c)^T$ in $\mathbf{R}^3$ and define a linear operator $T: \mathbf{R}^3 \to \mathbf{R}^3$ by the equation $T(\mathbf{x}) = \mathbf{v} \times \mathbf{x}$, the cross product of $\mathbf{v}$ and $\mathbf{x}$. Then $T(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbf{R}^3$ if $A =$

(A) $\begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & -c & b \\ c & 1 & -a \\ -b & a & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{pmatrix}$ (E) $\begin{pmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{pmatrix}$

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18. If A is an invertible matrix with an eigenvalue of 3 corresponding to the eigenvector $\mathbf{x}$, which of the following statements must be true?

(A) The matrix A^{-1} has an eigenvalue of $\frac{1}{3}$ corresponding to the eigenvector $\mathbf{x}$.

(B) The matrix A^{-1} has an eigenvalue of $\frac{1}{3}$ corresponding to the eigenvector whose entries are the reciprocals of the entries of $\mathbf{x}$.

(C) The matrix A^2 has an eigenvalue of 3 corresponding to the eigenvector $\mathbf{x}$.

(D) The matrix A^2 has an eigenvalue of 6 corresponding to the eigenvector $2\mathbf{x}$.

(E) The matrix A^2 has an eigenvalue of 3 corresponding to the eigenvector $3\mathbf{x}$.

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19. The eigenvalues of the matrix

$$A = \begin{pmatrix} 2 & b \\ 3 & -1 \end{pmatrix}$$

are -4 and $b - 1$. Find b.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

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20. The complex matrix

$$A = \begin{pmatrix} 2 & 2+i \\ 2-i & 6 \end{pmatrix}$$

has which one of the following as an eigenvalue?

(A) -1 (B) 3 (C) 7 (D) i (E) $1+i$
