

- 10 In order to send an undetected message to an agent in the field, each letter in the message is replaced by the number of its position in the alphabet and that number is entered in a matrix M . Thus, for example, "DEAD" becomes the matrix $M = \begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. In order to further avoid detection, each message with four letters is sent to the agent encoded as MC , where $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. If the agent receives the matrix $\begin{pmatrix} 51 & -3 \\ 31 & -8 \end{pmatrix}$, then the message is
- (A) RUSH (B) COME (C) ROME (D) CALL
(E) not uniquely determined by the information given

48. Let V be the set of all real polynomials $p(x)$. Let transformations T, S be defined on V by $T: p(x) \rightarrow xp(x)$ and $S: p(x) \rightarrow p'(x) = \frac{d}{dx}p(x)$, and interpret $(ST)(p(x))$ as $S(T(p(x)))$. Which of the following is true?

- (A) $ST = 0$
(B) $ST = I$
(C) $ST = TS$
(D) $ST = TS$ is the identity map of V onto itself
(E) $ST + TS$ is the identity map of V onto itself

15. If f is a linear transformation from the plane to the real numbers and if $f(1, 1) = 1$ and $f(-1, 0) = 2$, then $f(3, 5) =$
- (A) -6 (B) -5 (C) 0 (D) 8 (E) 9

38. If M is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then M^{100} is

- (A) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
(E) none of the above

35. The rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix} \text{ is}$$

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

32. The dimension of the subspace spanned by the real vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ is}$$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

52 Which of the following is the larger of the eigenvalues (characteristic values) of the matrix $\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$?

(A) 4

(B) 5

(C) 6

(D) 10

(E) 12

53 Let V be the vector space, under the usual operations, of real polynomials that are of degree at most 3. Let W be the subspace of all polynomials $p(x)$ in V such that $p(0) = p(1) = p(-1) = 0$. Then $\dim V + \dim W$ is

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8
