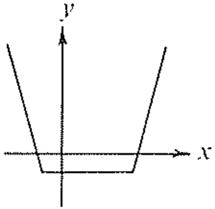
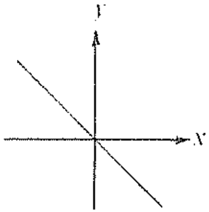
4. For $x \geqq 0$, $\frac{d}{dx}(x^e \cdot e^x) =$

(A) $x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$ (B) $x^e \cdot e^x + x^{e+1} \cdot e^{x-1}$ (C) $x^e \cdot e^x$ (D) $x^{e-1} \cdot e^{x+1}$ (E) $x^{e+1} \cdot e^{x-1}$

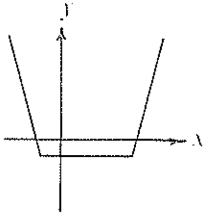


6. Which of the following could be the graph of the derivative of the function whose graph is shown in the figure above?

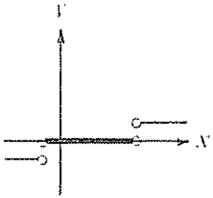
(A)



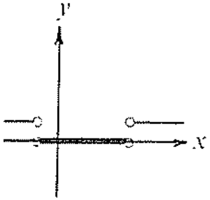
(B)



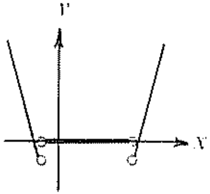
(C)

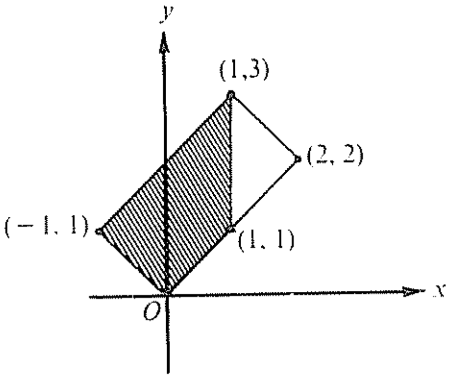


(D)



(E)





7. Which of the following integrals represents the area of the shaded portion of the rectangle shown in the figure above?

(A) $\int_{-1}^{1} (x + 2 - |x|)\,dx$ (B) $\int_{-1}^{1} (|x| + x + 2)\,dx$ (C) $\int_{-1}^{1} (x + 2)\,dx$

(D) $\int_{-1}^{1} |x|\,dx$ (E) $\int_{-1}^{1} 2\,dx$

---

8. $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

(A) $\frac{1}{e}$ (B) $\log 2$ (C) 1 (D) e (E) $+\infty$

11. If $\sin^{-1}x = \frac{\pi}{6}$, then the acute angle value of $\cos^{-1}x$ is

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\sqrt{1 - \frac{\pi^2}{6^2}}$ (D) $1 - \frac{\pi}{6}$ (E) 0

12. $\int_0^{\pi} e^{\sin^2 x} e^{\cos^2 x} dx =$

(A) π (B) $e\pi$ (C) e^{π} (D) $e^{\sin^2 \pi}$ (E) $e^{\pi} - 1$

---

13. Which of the following is true of the behavior of $f(x) = \dfrac{x^3 + 8}{x^2 - 4}$ as $x \to 2$?

(A) The limit is 0.
(B) The limit is 1.
(C) The limit is 4
(D) The graph of the function has a vertical asymptote at 2.
(E) The function has unequal, finite left-hand and right-hand limits

16. Suppose that an arrow is shot from a point p and lands at a point q such that at one and only one point in its flight is the arrow parallel to the line of sight between p and q. Of the following, which is the best mathematical model for the phenomenon described above?

(A) A function f differentiable on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f'(x)\,dx = c(b-a)$

(B) A function f whose second derivative is at all points negative such that there is one and only one point c in $[a, b]$ with $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

(C) A function f whose first derivative is at all points positive such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c)\cdot(b-a)$

(D) A function f continuous on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c)\cdot(b-a)$

(E) A function f continuous on $[a, b]$ and $f(a) < d < f(b)$ such that there is one and only one point c in $[a, b]$ with $f(c) = d$

19. If $c > 0$ and $f(x) = e^x - cx$ for all real numbers x, then the minimum value of f is

(A) $f(c)$ (B) $f(e^c)$ (C) $f\left(\frac{1}{c}\right)$ (D) $f(\log c)$ (E) nonexistent

20. Suppose that $f(1 + x) = f(x)$ for all real x. If f is a polynomial and $f(5) = 11$, then $f\left(\frac{15}{2}\right)$ is

(A) -11 (B) 0 (C) 11 (D) $\frac{33}{2}$

(E) not uniquely determined by the information given

---

21. For all $x > 0$, if $f(\log x) = \sqrt{x}$, then $f(x) =$

(A) $e^{\frac{x}{2}}$ (B) $\log\sqrt{x}$ (C) $e^{\sqrt{x}}$ (D) $\sqrt{\log x}$ (E) $\frac{\log x}{2}$

---

27 For what triples of real numbers (a, b, c) with $a \neq 0$ is the function

defined by $f(x) = \begin{cases} x, & \text{if } x \leqq 1 \\ ax^2 + bx + c, & \text{if } x > 1 \end{cases}$

differentiable at all real x ?

(A) $\{(a,\ 1 - 2a,\ a) \mid a$ is a nonzero real number$\}$

(B) $\{(a,\ 1 - 2a,\ c) \mid a, c$ are real numbers and $a \neq 0\}$

(C) $\{(a, b, c) \mid a, b, c$ are real numbers, $a \neq 0$, and $a + b + c = 1\}$

(D) $\left\{\left(\frac{1}{2}, 0, 0\right)\right\}$

(E) $\{(a,\ 1 - 2a,\ 0) \mid a$ is a nonzero real number$\}$

Questions 28-30 are based on the following information.

Let f be a function such that the graph of f is a semicircle S with end points $(a, 0)$ and $(b, 0)$ where $a < b$.

28 $\left|\int_a^b f(x)dx\right| =$

(A) $f(b) - f(a)$ (B) $\frac{f(b) - f(a)}{b - a}$ (C) $(b - a)\frac{\pi}{4}$ (D) $(b - a)^2\pi$ (E) $(b - a)^2\frac{\pi}{8}$

29. The graph of $y = 3f(x)$ is a

(A) translation of S (B) semicircle with radius three times that of S (C) subset of an ellipse

(D) subset of a parabola (E) subset of a hyperbola

30. The improper integral $\int_a^b f(x)f'(x)dx$ is

(A) necessarily zero
(B) possibly zero but not necessarily
(C) necessarily nonexistent
(D) possibly nonexistent but not necessarily
(E) none of the above

31. $\lim_{x \to \pi} \frac{e^{-\pi} - e^{-x}}{\sin x} =$

(A) $-\infty$ (B) $-e^{-\pi}$ (C) 0 (D) $e^{-\pi}$ (E) 1

36 The shortest distance from the curve $xy = 8$ to the origin is

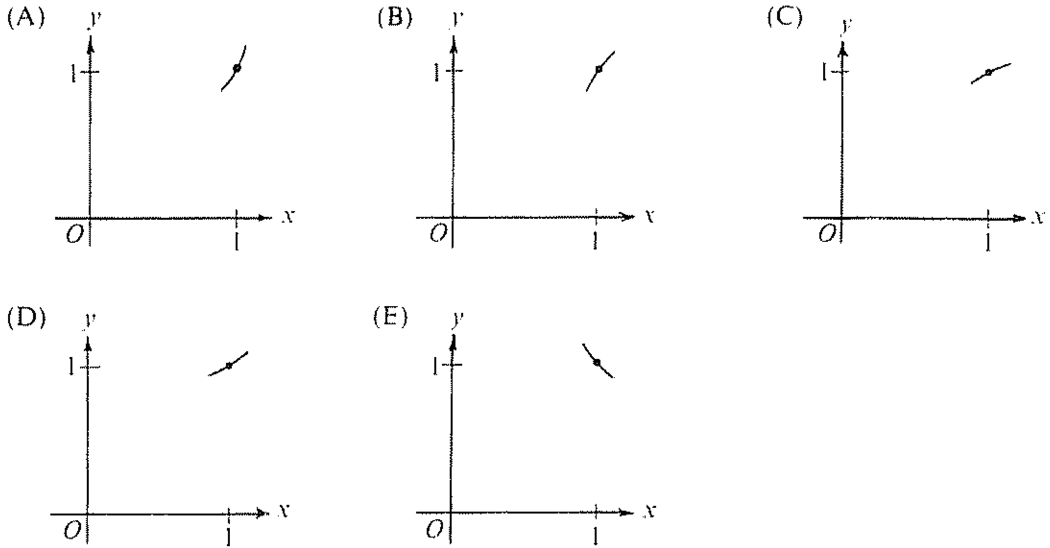
(A) 4 (B) 8 (C) 16 (D) $2\sqrt{2}$ (E) $4\sqrt{2}$

39. If $f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0, \end{cases}$ then $\int_{-1}^{1} f(x)\,dx$ is

(A) -2 (B) 0 (C) 2 (D) not defined

(E) none of the above

41. Of the following, which best represents a portion of the graph of $y = \frac{1}{e^x} + x - \frac{1}{e}$ near (1, 1)?



44. Suppose f is a real function such that $f'(x_0)$ exists. Which of the following is the value of

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0 - h)}{h} ?$$

(A) 0 (B) $2f'(x_0)$ (C) $f'(-x_0)$ (D) $-f'(x_0)$ (E) $-2f'(x_0)$

45. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{e^n}{n!} x^n$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) $+\infty$

---

46. In the xy-plane, the graph of $x^{\log y} = y^{\log x}$ is

(A) empty
(B) a single point
(C) a ray in the open first quadrant
(D) a closed curve
(E) the open first quadrant

51. Let $x_1 = 1$ and $x_{n+1} = \sqrt{3 + 2x_n}$ for all positive integers n. If it is assumed that $\{x_n\}$ converges, then $\lim_{n \to \infty} x_n =$

(A) -1

(B) 0

(C) $\sqrt{5}$

(D) e

(E) 3

56. The polynomial $p(x) = 1 + \frac{1}{2}(x - 1) - \frac{1}{8}(x - 1)^2$ is used to approximate $\sqrt{1.01}$. Which of the following most closely approximates the error $\sqrt{1.01} - p(1.01)$?

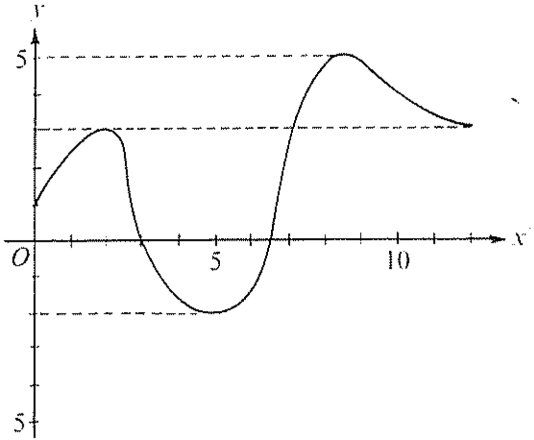
(A) $\left(\frac{1}{16}\right) \times 10^{-6}$

(B) $\left(\frac{1}{48}\right) \times 10^{-8}$

(C) $\left(\frac{3}{8}\right) \times 10^{-10}$

(D) $-\left(\frac{3}{8}\right) \times 10^{-10}$

(E) $-\left(\frac{1}{16}\right) \times 10^{-6}$



59. If f is the function whose graph is indicated in the figure above, then the least upper bound (supremum) of

$$\left\{ \sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| : 0 = x_0 < x_1 < \qquad < x_{n-1} < x_n = 12 \right\}$$

appears to be

(A) 2 (B) 7 (C) 12 (D) 16 (E) 21

63. Let f be a continuous, strictly decreasing, real-valued function such that $\int_0^{+\infty} f(x)\,dx$ is finite and $f(0) = 1$

In terms of f^{-1} (the inverse function of f), $\int_0^{+\infty} f(x)\,dx$ is

(A) less than $\int_1^{+\infty} f^{-1}(y)\,dy$

(B) greater than $\int_0^1 f^{-1}(y)\,dy$

(C) equal to $\int_1^{+\infty} f^{-1}(y)\,dy$

(D) equal to $\int_0^1 f^{-1}(y)\,dy$

(E) equal to $\int_0^{+\infty} f^{-1}(y)\,dy$

WORK SHEET for the MATHEMATICS Test, Form GR8767 ONLY
Answer Key and Percentage* of Examinees Answering Each Question Correctly

QUESTION Number	Answer	P+	TOTAL C	I
1	B	92		
2	C	72		
3	B	94		
4	A	89		
5	A	89		
6	C	83		
7	A	81		
8	E	76		
9	E	84		
10	C	79		
11	B	77		
12	B	81		
13	D	82		
14	A	47		
15	E	77		
16	B	61		
17	C	49		
18	D	65		
19	D	71		
20	C	42		
21	A	64		
22	B	54		
23	C	56		
24	D	80		
25	D	53		
26	E	54		
27	A	34		
28	E	78		
29	C	58		
30	A	29		
31	B	58		
32	B	62		
33	E	41		
34	A	51		
35	B	29		
36	A	54		
37	D	38		
38	A	69		
39	B	63		
40	C	30		

Correct (C) ______

Incorrect (I) ______

QUESTION Number	Answer	P+	TOTAL C	I
41	D	47		
42	D	33		
43	E	49		
44	B	57		
45	E	46		
46	E	42		
47	D	48		
48	D	67		
49	C	41		
50	D	40		
51	E	52		
52	C	59		
53	B	23		
54	E	39		
55	C	16		
56	A	31		
57	C	46		
58	B	37		
59	D	35		
60	C	23		
61	C	37		
62	B	33		
63	D	40		
64	C	39		
65	A	48		
66	B	57		

Correct (C) ______

Incorrect (I) ______

Total Score

C − I/4 = ______

Scaled Score (SS) = ______

*Estimated P+ for the group of examinees who took the GRE Mathematics Test in a recent three-year period