MULTIVARIABLE CALCULUS

*** from test 2

3 $\int_0^1 \int_0^x xy \, dy \, dx =$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{3}$ (D) 1 (E) 3

5. All functions f defined on the xy-plane such that

$$\frac{\partial f}{\partial x} = 2x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x + 2y$$

are given by $f(x, y) =$

(A) $x^2 + xy + y^2 + C$ (B) $x^2 - xy + y^2 + C$ (C) $x^2 - xy - y^2 + C$

(D) $x^2 + 2xy + y^2 + C$ (E) $x^2 - 2xy + y^2 + C$

22 $\int_0^1 \left(\int_0^{\sin y} \frac{1}{\sqrt{1 - x^2}} \, dx \right) dy =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) 1 (E) $\frac{\pi}{3}$

42. In xyz-space, the degree measure of the angle between the rays

$$z = x \geqq 0, \; y = 0$$

and

$$z = y \geqq 0, \; x = 0$$ is

(A) 0° (B) 30° (C) 45° (D) 60° (E) 90°

*** from test 3

11. If $\phi(x, y, z) = x^2 + 2xy + xz^{\frac{3}{2}}$, which of the following partial derivatives are identically zero?

I. $\frac{\partial^2 \phi}{\partial y^2}$

II. $\frac{\partial^2 \phi}{\partial x \partial y}$

III. $\frac{\partial^2 \phi}{\partial z \partial y}$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

20. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?

(A) $4\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(B) $\int_{x=-2}^{x=2} \int_{y=-\sqrt{(4-x^2)/2}}^{y=\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(C) $4\int_{y=0}^{y=\sqrt{2}} \int_{x=-\sqrt{4-2y^2}}^{x=\sqrt{4-2y^2}} dx\, dy$

(D) $\int_{y=-\sqrt{2}}^{y=\sqrt{2}} \int_{x=-2}^{x=2} (8 - 2x^2 - 4y^2)\, dx\, dy$

(E) $2\int_{y=0}^{y=\sqrt{2}} \int_{x=0}^{x=\sqrt{4-2y^2}} (8 - 2x^2 - 4y^2)\, dx\, dy$

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26. Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v}_2 =$

(A) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ (B) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (C) $\mathbf{i} + \mathbf{k}$ (D) $\mathbf{j} - \mathbf{k}$ (E) $\mathbf{i} + \mathbf{j}$

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27. If the curve in the yz-plane with equation $z = f(y)$ is rotated around the y-axis, an equation of the resulting surface of revolution is

(A) $x^2 + z^2 = [f(y)]^2$
(B) $x^2 + z^2 = f(y)$
(C) $x^2 + z^2 = |f(y)|$
(D) $y^2 + z^2 = |f(y)|$
(E) $y^2 + z^2 = [f(x)]^2$

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47. Let C be the ellipse with center $(0, 0)$, major axis of length $2a$, and minor axis of length $2b$. The value of $\oint_C x\,dy - y\,dx$ is

(A) $\pi\sqrt{a^2 + b^2}$

(B) $2\pi\sqrt{a^2 + b^2}$

(C) $2\pi ab$

(D) πab

(E) $\frac{\pi ab}{2}$

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53. Let $r > 0$ and let C be the circle $|z| = r$ in the complex plane. If P is a polynomial function, then $\int_C P(z)\,dz =$

(A) 0
(B) πr^2
(C) $2\pi i$
(D) $2\pi P(0)i$
(E) $P(r)$

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63. Let R be the circular region of the xy-plane with center at the origin and radius 2. Then $\int_R\int e^{-(x^2+y^2)}dx\,dy =$

(A) 4π
(B) πe^{-4}
(C) $4\pi e^{-4}$
(D) $\pi(1 - e^{-4})$
(E) $4\pi(e - e^{-4})$

*** from test 1

26. Let $f(x, y) = x^2 - 2xy + y^3$ for all real x and y. Which of the following is true?

(A) f has all of its relative extrema on the line $x = y$.

(B) f has all of its relative extrema on the parabola $x = y^2$.

(C) f has a relative minimum at $(0, 0)$.

(D) f has an absolute minimum at $\left(\frac{2}{3}, \frac{2}{3}\right)$.

(E) f has an absolute minimum at $(1, 1)$.

34. The minimal distance between any point on the sphere $(x-2)^2 + (y-1)^2 + (z-3)^2 = 1$ and any point on the sphere $(x+3)^2 + (y-2)^2 + (z-4)^2 = 4$ is

(A) 0 (B) 4 (C) $\sqrt{27}$ (D) $2(\sqrt{2}+1)$ (E) $3(\sqrt{3}-1)$

41. Let C be the circle $x^2 + y^2 = 1$ oriented counterclockwise in the xy-plane. What is the value of the line integral $\oint_C (2x - y)\,dx + (x + 3y)\,dy$?

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) π (E) 2π

Test 2

(3) B $\left(\frac{1}{8}\right)$

(5) A $x^2 + xy + y^2 + C$

(22) B $\frac{1}{2}$

(42) D 60°

Test 3

(11) (C) (i) + (iii)

(20) (B) $V = \int_{-2}^{2} \int_{-\sqrt{(4-x^2)/2}}^{+\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(26) (C) $v_1 = i + j - k$, $v_2 = i + k$

(27) (A) $x^2 + z^2 = [f(y)]^2$

(47) (C) $2\pi ab$

(53) (A) 0

(63) (D) $\pi(1 - e^{-4})$

Test 1

(26) A (on line $x = y$)

(34) E $(= 3(\sqrt{3} - 1))$

(41) E $(= 2\pi)$